CBSE Board

Class XII Mathematics

Sample Paper - 1

Term 2 – 2021-22

Time: 2 hours **Total Marks: 40**

General Instructions:

1. *This question paper contains three sections – A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each*
3. *Section – B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

Section A

Q1 – Q6 are of 2 marks each.

1. Integrate $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$

OR

Integrate $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1}+\sqrt[3]{11-x}} dx$

2. Check whether the differential equation $2ye^{x/y}dx + \left(y - 2x\, e^{x/y}\right)dy = 0$ is homogeneous.

3. Write the direction cosines of the vectors $-2\hat{i} + \hat{j} - 5\hat{k}$.

4. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane 10x + 2y – 11z = 3

5. Bag A contains 3 white and 4 red balls, and bag A contains 6 white and 3 red balls. An unbiased coin, twice as likely to come up heads as tails, is tossed once. If it shows head, a ball drawn from bag A, otherwise, from bag B. Given that a white ball was drawn, what is the probability that the coin came up tail?

6. When 150 identical coins are tossed the probability that the coins will show head is p. Suppose the probability of showing heads on 75 coins is equal to the probability of showing heads on 76 coins with the condition, $0 < p < 1$, then find the value of p.

Section B

Q7 – Q10 are of 3 marks each

7. Integrate: $\int \frac{dx}{3 - 10x - 25x^2}$

8. Solve the given differential equation $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$ if y(0) = 0.

OR

Find the particular solution of this differential equation $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$ if it is homogeneous, given that x = 0 when y = 1.

9. Find λ if the vectors $\vec{a} = \hat{i} - \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are perpendicular to each other

10. The vector equations of two lines are:

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} - 3\hat{j} + \hat{k})$

Find the shortest distance between the above lines.

OR

Find the vector and Cartesian equation of the plane through $3\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the lines

$\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$

$\vec{r} = \hat{i} - 3\hat{j} + \hat{k} + \mu(-5\hat{i} + 4\hat{j})$

Section C

Q11 – Q14 are of 4 marks each

11. Integrate $\int(3x-2)\sqrt{x^2+x+1}dx$

12. Using integration, find the area between the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

OR

Find the area of the smaller region bounded by the ellipse
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$

13. Find the vector equation of the plane passing through three points with position vector $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\ 2\hat{i}-2\hat{j}+\hat{k}$.

14. Case Study
Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

i. All the four cards are spades?
ii. Only 3 cards are spades?

SPECTRA PATH TO SUCCESS

Solution

Section A

1. $I=\int \frac{\cos 2x}{\sin^2 x\cos^2 x}dx$

$I=\int \frac{\cos^2 x-\sin^2 x}{\sin^2 x\cos^2 x}dx$

$I=\int \left(\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}\right)dx$

$I=\int \left(\operatorname{cosec}^2 x-\sec^2 x\right)dx$

$I=-\cot x-\tan x+c$

OR

$I=\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1}+\sqrt[3]{11-x}}dx \ldots(i)$

$I=\int_2^8 \frac{\sqrt[3]{10-x+1}}{\sqrt[3]{10-x+1}+\sqrt[3]{11-(10-x)}}dx$

$I=\int_2^8 \frac{\sqrt[3]{11-x}}{\sqrt[3]{11-x}+\sqrt[3]{1+x}}dx \ldots(ii)$

Adding (i) and (ii)

$2I=\int_2^8 \frac{\sqrt[3]{x+1}+\sqrt[3]{11-x}}{\sqrt[3]{x+1}+\sqrt[3]{11-x}}dx$

$I=\frac{1}{2}\int_2^8 dx$

$I=\frac{1}{2}[x]_2^8$

I = 3

SPECTRA PATH TO SUCCESS

2. Given DE is $2ye^{x/y}dx + \; y - 2x\, e^{x/y} \; dy = 0$

$$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}} \qquad \ldots 1$$

Let

$$F \; x, y \; = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}$$

Then,

$$F \; \lambda x, \lambda y \; = \frac{\lambda\left(2xe^{\frac{x}{y}} - y\right)}{\lambda\left(2ye^{\frac{x}{y}}\right)} = \lambda^{o}\left[F \; x, y \;\right]$$

Thus, F(x, y) is a homogeneous function of degree zero.

3. Let $\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$, then

$$\left|\vec{a}\right| = \sqrt{(-2)^2 + (1)^2 + (-5)^2}$$

$$\Rightarrow \left|\vec{a}\right| = \sqrt{4 + 1 + 25}$$

$$\Rightarrow \left|\vec{a}\right| = \sqrt{30}$$

$$\text{Now}, \hat{a} = \frac{\vec{a}}{\left|\vec{a}\right|} = \frac{-2\hat{i} + \hat{j} - 5\hat{k}}{\sqrt{30}} = -\frac{2}{\sqrt{30}}\hat{i} + \frac{1}{\sqrt{30}}\hat{j} - \frac{5}{\sqrt{30}}\hat{k}$$

Thus, the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$ are $-\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}$ and $-\frac{5}{\sqrt{30}}$.

4. Let ϕ be the angle between the line and plane and θ be the angle between the line and normal to plane

$\phi = (90 - \theta)$

Let $\vec{b}$ be vector parallel to line

$$\Rightarrow \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Let $\hat{n}$ be normal to plane

$$\Rightarrow \hat{n} = 10\hat{i} + 2\hat{j} - 11\hat{k}$$

$$\cos\theta = \left|\frac{\left(2\hat{i}+3\hat{j}+6\hat{k}\right).\left(10\hat{i}+2\hat{j}-11\hat{k}\right)}{\sqrt{2^2+3^2+6^2}\ \sqrt{10^2+2^2+11^2}}\right|$$

$$\Rightarrow \cos(90-\phi) = \left|\frac{-40}{7\times 15}\right| = \left|\frac{-8}{21}\right|$$

$$\Rightarrow \sin\phi = \frac{8}{21} \Rightarrow \phi = \sin^{-1}\left(\frac{8}{21}\right)$$

5.

Let W be the white ball was drawn and T be the tail come up.

$$P\left(\frac{T}{W}\right) = \frac{P(T\cap W)}{P(W)}$$

$$= \frac{\frac{1}{2}\times\frac{6}{9}}{\frac{1}{2}\times\frac{6}{9}+\frac{1}{2}\times\frac{3}{7}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}+\frac{3}{14}}$$

$$= \frac{14}{17}$$

6. Let X be the number of coins showing heads.

Let X be a binomial variate with parameter $n = 150$ and p.

$$P(X = 75) = P(X = 76)$$

$$\Rightarrow C_{75}^{150}p^{75}(1-p)^{75} = C_{76}^{150}p^{76}(1-p)^{74}$$

$$\Rightarrow \frac{150!}{75!\,75!}(1-p) = \frac{150!}{76!\,74!}p$$

$$\Rightarrow \frac{1}{75}(1-p) = \frac{1}{76}p$$

$$\Rightarrow \frac{p}{1-p} = \frac{76}{75}$$

$\Rightarrow 75p = 76 - 76p$

$\Rightarrow 151p = 76$

$\Rightarrow p = \dfrac{76}{151}$

Section B

7. $I = \int \dfrac{dx}{3 - 10x - 25x^2}$

$I = -\dfrac{1}{25}\int \dfrac{dx}{x^2 + \frac{10}{25}x - \frac{3}{25}}$

$I = -\dfrac{1}{25}\int \dfrac{dx}{x^2 + \frac{10}{25}x - \frac{3}{25}}$

$I = -\dfrac{1}{25}\int \dfrac{dx}{x^2 + \frac{10}{25}x + \frac{1}{25} - \frac{3}{25} - \frac{1}{25}}$

$I = -\dfrac{1}{25}\int \dfrac{dx}{\left(x + \frac{1}{5}\right)^2 - \left(\frac{2}{5}\right)^2}$

$I = -\dfrac{1}{25} \times \dfrac{1}{2 \times \frac{2}{5}} \log\left|\dfrac{5x + 1 - 2}{5x + 1 + 2}\right| + c$

$I = -\dfrac{1}{20}\log\left|\dfrac{5x - 1}{5x + 3}\right| + c$

8. Given DE is $(x + 1)\dfrac{dy}{dx} = 2e^{-y} - 1$ ... (1)

$\Rightarrow \dfrac{dy}{2e^{-y} - 1} = \dfrac{dx}{(x + 1)}$

$\Rightarrow \int \dfrac{dy}{2e^{-y} - 1} = \int \dfrac{dx}{(x + 1)}$

$\Rightarrow -\int -\dfrac{e^{y}dy}{2 - e^{y}} = \int \dfrac{dx}{(x + 1)}$

SPECTRA
PATH TO SUCCESS

$$-\log\left|2-e^{y}\right| = \log|x+1| + c$$

$$\Rightarrow \log\left|(x+1)\left(2-e^{y}\right)\right| = -c$$

$$\Rightarrow \left|(x+1)\left(2-e^{y}\right)\right| = e^{-C}$$

$$\Rightarrow (x+1)\left(2-e^{y}\right) = \pm e^{-C} = A \text{ ... (say)}$$

$$\Rightarrow (x+1)\left(2-e^{y}\right) = A \text{ ... (ii)}$$

$x = 0, y = 0$

$$(0+1)\left(2-e^{0}\right) = A$$

$$\Rightarrow 1(2-1) = A$$

$$\Rightarrow A = 1$$

Substituting in (ii) , we get

$$(x+1)\left(2-e^{y}\right) = 1$$

OR

Let x = vy

Differentiating w.r.t. y, we get

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

Substituting the value of x and $\frac{dx}{dy}$ in equation (1), we get

$$v + y\frac{dv}{dy} = \frac{2vye^{v} - y}{2ye^{v}} = \frac{2ve^{v} - 1}{2e^{v}}$$

$$y\frac{dv}{dy} = \frac{2ve^{v} - 1}{2e^{v}} - v$$

$$y\frac{dv}{dy} = -\frac{1}{2e^{v}}$$

$$2e^{v}dv = \frac{-dy}{y}$$

$$\int 2e^{v}.dv = -\int \frac{dy}{y}$$

$2e^{v} = -\log|y| + C$

Substituting the value of v, we get

$$2e^{\frac{x}{y}} + \log|y| = C \qquad \text{... 2}$$

Substituting x = 0 and y = 1 in equation (2), we get

$$2e^{o} + \log|1| = C \Rightarrow C = 2$$

Substituting the value of C in equation (2), we get

$2e^{\frac{x}{y}} + \log|y| = 2,$ Which is the particular solution of the given differential equation.

9.

Let $\hat{n}$ be the unit vector along the sum of vectors $\vec{b} + \vec{c}$:

$$\hat{n} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

The scalar product of $\vec{a}$ and $\hat{n}$ is 1. Thus,

$$\vec{a} \cdot \hat{n} = \left(\hat{i} + \hat{j} + \hat{k}\right) \cdot \left(\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}\right)$$

$$\Rightarrow 1 = \frac{1(2+\lambda) + 1\cdot 6 - 1\cdot 2}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = 2 + \lambda + 6 - 2$$

$$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = \lambda + 6$$

$$\Rightarrow (2+\lambda)^2 + 40 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 4 + 40 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 4\lambda + 44 = 12\lambda + 36$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

10. The given equations of the lines are

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} - 3\hat{j} + \hat{k}\right)$

On comparing with standard equation of line

We have $a_1 = \hat{i} + 2\hat{j} + 3\hat{k},\ b_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$a_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}, b_2 = 2\hat{i} - 3\hat{j} + \hat{k}$

Therefore, $a_2 - a_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

SPECTRA PATH TO SUCCESS

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\therefore \left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right) = \left(3\hat{i} + 3\hat{j} + 3\hat{k}\right).\left(-9\hat{i} + 3\hat{j} + 9\hat{k}\right)$$

$$= (3)(-9) + 3(3) + 3(9)$$

$$= -27 + 9 + 27 = 9$$

And, $\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-9)^2 + 3^2 + 9^2}$

$$= \sqrt{81 + 9 + 81} = \sqrt{171} = 3\sqrt{19}$$

$$\therefore \text{Shortest Distance } = \left|\frac{\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

OR

Required plane passes through the point with position vector $\vec{r} = 3\hat{i} - \hat{j} + 2\hat{k}$ and parallel to vectors $\vec{b} = 2\hat{i} - 5\hat{j} - \hat{k}$ and $\vec{d} = -5\hat{i} + 4\hat{j}$

The equation of the plane is,

$$\left(\vec{r} - \vec{r}_1\right).\left(\vec{b} \times \vec{d}\right) = 0$$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{vmatrix} = 4\hat{i} + 5\hat{j} - 17\hat{k}$$

So the equation is,

$$\left(\vec{r} - \left(3\hat{i} - \hat{j} + 2\hat{k}\right)\right).\left(4\hat{i} + 5\hat{j} - 17\hat{k}\right) = 0$$

$$\Rightarrow \vec{r}.\left(4\hat{i} + 5\hat{j} - 17\hat{k}\right) - \left(3.4 + (-1).5 + 2(-17)\right) = 0$$

$$\Rightarrow \vec{r}.\left(4\hat{i} + 5\hat{j} - 17\hat{k}\right) + 27 = 0 \text{[Vector equation]}$$

$$\Rightarrow 4x + 5y - 17y + 27 = 0 \quad \text{[Cartesian equation]}$$

Section C

11. $I=\int(3x-2)\sqrt{x^2+x+1}dx$

We express

$$3x-2=A\frac{d}{dx}\left(x^2+x+1\right)+B$$

$$\Rightarrow 3x-2=A\left(2x+1\right)+B$$

$$\Rightarrow 3x-2=2Ax+A+B$$

⇒ 2A = 3 i.e. A = 3/2

⇒ A + B = -2

⇒ B = -7/2

$$I=\int\left(\frac{3}{2}(2x+1)-\frac{7}{2}\right)\sqrt{x^2+x+1}dx$$

$$I=\int\left(\frac{3}{2}(2x+1)\sqrt{x^2+x+1}\right)dx-\int\frac{7}{2}\sqrt{x^2+x+1}dx$$

$$I=I_1-I_2$$

$$I_1=\int\left(\frac{3}{2}(2x+1)\sqrt{x^2+x+1}\right)dx$$

Put $t = x^2 + x + 1 \Rightarrow (2x + 1)dx = dt$

$$I_1=\frac{3}{2}\int\sqrt{t}dt=\frac{3}{2}\times\frac{t^{\frac{3}{2}}}{\frac{3}{2}}=t^{\frac{3}{2}}+c=\sqrt[3]{x^2+x+1}+c$$

$$I_2=\int\frac{7}{2}\sqrt{x^2+x+1}dx$$

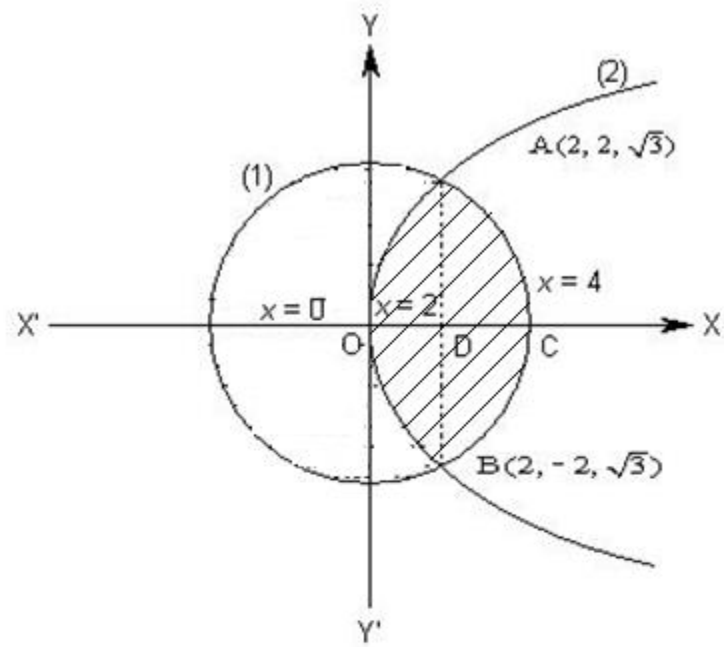
$$I_2=\frac{7}{2}\int\sqrt{x^2+x+\frac{1}{4}+1-\frac{1}{4}}dx$$

$$I_2=\frac{7}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx=\frac{7}{4}\left(x+\frac{1}{2}\right)\sqrt{x^2+x+1}+\frac{3}{8}\log\left|x+\sqrt{x^2+x+1}\right|+c$$

$$I=\sqrt[3]{x^2+x+1}+\frac{7}{4}\left(x+\frac{1}{2}\right)\sqrt{x^2+x+1}+\frac{3}{8}\log\left|x+\sqrt{x^2+x+1}\right|+c$$

SPECTRA PATH TO SUCCESS

12. $x^2 + y^2 = 16$ … (i) and $y^2 = 6x$ … (ii)



Points of intersection of curve (i) and (ii) is

$\therefore$ $A\left(2, 2\sqrt{3}\right)$ and $B\left(2, -2\sqrt{3}\right)$

Also $C(4,0)$.

Area OBCAO = 2 (Area ODA + Area DCA)

$$= 2\left[\int_0^2 y_2 dx + \int_2^4 y_1 dx\right]$$

$$= 2\left[\int_0^2 \sqrt{6x}\, dx + \int_2^4 \sqrt{16 - x^2}\, dx\right]$$

$$= 2\left[\sqrt{6}.\left\{\frac{2}{3}x^{3/2}\right\}_0^2 + \left\{\frac{x\sqrt{16 - x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right\}_2^4\right]$$

$$=2\left[\frac{2\sqrt{6}}{3}.2\sqrt{2}+\left\{0+8\sin^{-1}1\right\}-\left\{\frac{2.2\sqrt{3}}{2}+8\sin^{-1}\frac{1}{2}\right\}\right]$$

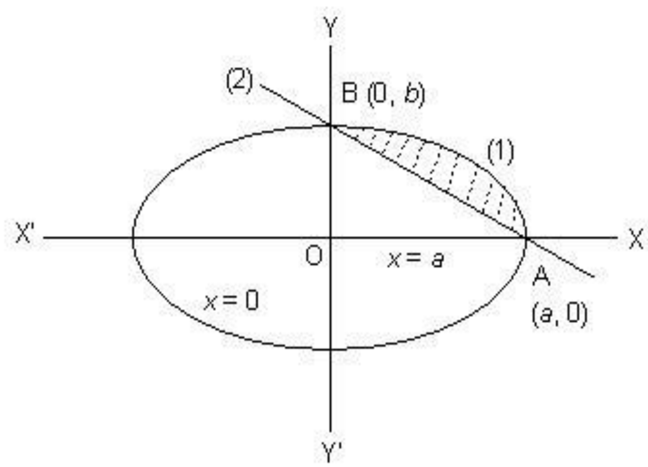
$$=\frac{16\sqrt{3}}{3}+16.\frac{\pi}{2}-\left(4\sqrt{3}+16.\frac{\pi}{6}\right)$$

$$=\left(\frac{4\sqrt{3}}{3}+\frac{16}{3}\pi\right)\text{sq. units.}$$

OR

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(1)$$

$$\frac{x}{a}+\frac{y}{b}=1 \qquad ...(2)$$



Equation (i) gives y = $b\sqrt{1-\frac{x^2}{a^2}}$

Equation (ii) gives y = $b\left(1-\frac{x}{a}\right)$

(Area of the smaller region)

$$=\int_0^a\left(y_1-y_2\right)dx$$

$$= \int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx - \int_0^a b\left(1-\frac{x}{a}\right)dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}dx - b\int_0^a \left(1-\frac{x}{a}\right)dx$$

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - b\left[x-\frac{x^2}{2a}\right]_0^a$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1}1\right] - b\left[a-\frac{a^2}{2a}\right]$$

$$= \frac{ab}{2}.\frac{\pi}{2}-\frac{ab}{2} = \frac{1}{4}ab(\pi-2) \text{ sq. units.}$$

13. Let the position vectors of the three points be,
$\vec{a}=\hat{i}+\hat{j}-2\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}+\hat{k}$.

So, the equation of the plane passing through the points $\vec{a}, \vec{b}$ and $\vec{c}$ is

$$(\vec{r}-\vec{a}).\left[(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\right]=0$$

$$\Rightarrow \left[\vec{r}-(\hat{i}+\hat{j}-2\hat{k})\right].\left[(\hat{i}-3\hat{j})\times(\hat{j}+3\hat{k})\right]=0$$

$$\Rightarrow \left[\vec{r}-(\hat{i}+\hat{j}-2\hat{k})\right].(\hat{k}-3\hat{j}-9\hat{i})=0$$

$$\Rightarrow \vec{r}.(-9\hat{i}-3\hat{j}+\hat{k})+14=0$$

$$\Rightarrow \vec{r}.(9\hat{i}+3\hat{j}-\hat{k})=14 \qquad \ldots(1)$$

So, the vector equation of the required plane is $\vec{r}.(9\hat{i}+3\hat{j}-\hat{k})=14$.

The equation of the given line is $\vec{r}=(3\hat{i}-\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$.

Position vector of any point on the give line is
$\vec{r}=(3+2\lambda)\hat{i}+(-1-2\lambda)\hat{j}+(-1+\lambda)\hat{k} \qquad \ldots(2)$

The point (2) lies on plane (1) if,

$$\left[(3+2\lambda)\hat{i}+(-1-2\lambda)\hat{j}+(-1+\lambda)\hat{k}\right].(9\hat{i}+3\hat{j}-\hat{k})=14$$

$$\Rightarrow 9(3+2\lambda)+3(-1-2\lambda)-(-1+\lambda)=14$$

$$\Rightarrow 11\lambda+25=14$$

$$\Rightarrow \lambda=-1$$

Putting $\lambda=-1$ in (2), we have

$\vec{r} = \; 3+2\lambda \; \hat{i} + \; -1-2\lambda \; \hat{j} + \; -1+\lambda \; \hat{k}$

$= \; 3+2 \; -1 \;\; \hat{i} + \; -1-2 \; -1 \;\; \hat{j} + \; -1+ \; -1 \;\; \hat{k}$

$= \hat{i} + \hat{j} - 2\hat{k}$

Thus, the position vector of the point of intersection of the given line and plane (1) is $\hat{i} + \hat{j} - 2\hat{k}$ and its co-ordinates are $1, 1, -2$.

14.

This is a case of bernoulli trials.

p = P(Success) = P(getting a spade in a single draw) $= \frac{13}{52} = \frac{1}{4}$

$q = P(\text{Failure}) = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

i. All the four cards are spades = P(X = 4) = ${}^{4}C_{4}p^{4}q^{0} = \left(\frac{1}{4}\right)^{4} = \frac{1}{256}$

ii. Only 3 cards are spades= P(X = 3) = ${}^{4}C_{3}p^{3}q^{1} = \frac{12}{256} = \frac{3}{64}$

SPECTRA PATH TO SUCCESS