Sample Question Paper - 4

Mathematics (041)

Class- XII, Session: 2021-22

TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Evaluate: $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$ **[2]**

OR

Evaluate: $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}dx$

2. Solve the initial value problem: $(xe^{y/x} + y)\, dx = x\, dy$, $y(1) = 1$ **[2]**

3. For what value of λ are the vectors $\vec{a}$ and $\vec{b}$ perpendicular to each other? Where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$ **[2]**

4. Find the direction cosines of the line $\frac{x-2}{2} = \frac{2y-5}{-3}$, z = -1. Also, find the vector equation of the line. **[2]**

5. An experiment succeeds twice as often as it fails. Find the probability that in the next six trails, there will be at least 4 successes. **[2]**

6. A bag contains 4 red and 5 black balls, a second bag contains 3 red and 7 black balls. One ball is drawn at random from each bag, find the probability that the balls are of the same colour. **[2]**

Section B

7. Evaluate $\int \frac{2x+1}{\sqrt{3x+2}}dx$ **[3]**

8. Solve the following differential equation. **[3]**
$\cos^2 x \frac{dy}{dx} + y = \tan x$

OR

Form the differential equation of the family of circles touching the y - axis at the origin.

9. For any two vectors $\vec{a}$ and $\vec{b}$ prove that: $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left(|\vec{a}|^2 + |\vec{b}|^2\right)$. **[3]**

10. A line makes angles α, β, γ and δ with the diagonals of a cube, prove that **[3]**
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

OR

Find the shortest distance between the lines whose vector equations are
$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

Section C

11. Evaluate: $\int \frac{dx}{\sin x(3+2\cos x)}$. **[4]**

12. Draw a rough sketch of the region $f(x, y) : y^2 \leq 5x, 5x^2 + 5y^2 \leq 36\}$ and find the area enclosed by the region using method of integration. **[4]**

OR

Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

13. Find the shortest distance between the given lines. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$, $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$ **[4]**

CASE-BASED/DATA-BASED

14. In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03. **[4]**

Based on the above information, answer the following questions.

i. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Govind is

ii. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$?

Solution

MATHEMATICS BASIC 041

Class 12 - Mathematics

Section A

1. Let I = $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

   Also let x = tan θ then dx = $\sec^2\theta$ dθ

   I = $\int \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2 x}\right)$ $\sec^2\theta$ dθ

   = $\int \tan^{-1}(\tan 3\theta) \sec^2\theta$ dθ

   = $\int 3\theta \sec^2\theta$ dθ

   = 3[$\theta \int \sec^2\theta$ dθ - $\int$(1$\int \sec^2\theta$ dθ} dθ]

   = 3[θ tan θ - $\int$tan θ dθ]

   = 3[θ tan θ + log sec θ] + C

   = 3[θ $\tan^{-1}$ x - log $\sqrt{1+x^2}$] + C

   I = 3x[$\tan^{-1}$ x - $\frac{3}{2}$ log|1 + x^2] + C

OR

   Let I = $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}dx$ ...(i)

   Also let $\sqrt{x}$ = t then, we have

   d($\sqrt{x}$) = dt

   $\Rightarrow \quad \frac{1}{2\sqrt{x}}dx = dt$

   $\Rightarrow \quad dx = 2\sqrt{x}dt$

   $\Rightarrow$ dx = 2t dt [$\because$ $\sqrt{x}$ = t]

   Putting $\sqrt{x}$ = t and dx = 2t dt in equation (i) we get

   $I = \int \frac{\sec^2 t}{t} \times 2tdt$

   = 2$\int \sec^2$ t dt

   = 2 tan t + c

   = 2 tan $\sqrt{x}$ + c

   $\therefore$ I = 2 tan $\sqrt{x}$ + c

2. We have, ($xe^{y/x}$ + y) dx = x dy

   $\Rightarrow \frac{dy}{dx} = e^{y/x} + \frac{y}{x}$

   This is a homogeneous differential equation. Putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ it reduces to

   $v + x\frac{dv}{dx}$ = e^v + v

   $\Rightarrow x\frac{dv}{dx}$ = e^v

   $\Rightarrow$ e^{-v} dv = $\frac{dx}{x}$, if x $\neq$ 0

   Integrating both side with respect to x

   $\Rightarrow \int e^{-v}dv = \int \frac{1}{x}dx$

   $\Rightarrow$ -e^{-v} = log |x| + C

   $\Rightarrow$ -$e^{-y/x}$ = log |x| + C ...(i)

   It is given that y (1) = 1 i.e. when x = 1, y = 1. Putting x = 1, y = 1 in (i), we get: -e^{-1} = C

   Putting C = $-\frac{1}{e}$ in (i), we get

   -$e^{-y/x}$ = log |x| $-\frac{1}{e}$

   $\Rightarrow$ $e^{-y/x}$ = $\frac{1}{e}$ - log |x| $\Rightarrow -\frac{y}{x}$ = log (1 - e log |x|) = -1 $\Rightarrow$ y = x - x log (1 - e log |x|)

   Hence, y = x - x log (1 - e log |x|), is the solution of the given equation.

3. Since, $\vec{a}\ and\ \vec{b}$ are perpendicular

$\therefore \vec{a}.\vec{b} = 0$

$\Rightarrow$(2i + 3j + 4k).(3i + 2j - λk) = 0

$\Rightarrow$(2)(3) + (3)(2) + (4)(-λ) = 0

$\Rightarrow$6 + 6 - 4λ = 0

$\Rightarrow 12 - 4\lambda = 0$

$\Rightarrow -4\lambda = -12$

$\Rightarrow \lambda = \frac{-12}{-4}$

$\Rightarrow \lambda = 3$

4. The Cartesian equations of the given line are

$\frac{x-2}{2} = \frac{2y-5}{-3}$, z = -1

These above equations can be re-written as

$\frac{x-2}{2} = \frac{2y-5}{-3} = \frac{z+1}{0}$ or, $\frac{x-2}{2} = \frac{y-5/2}{-3/2} = \frac{z+1}{0}$

This shows that the given line passes through the point (2, $\frac{5}{2}$, - 1) and has direction ratios proportional to 2, -$\frac{3}{2}$,0. So, its direction cosines are

$\frac{2}{\sqrt{2^2+\left(-\frac{3}{2}\right)^2+0^2}}, \frac{-3/2}{\sqrt{2^2+\left(-\frac{3}{2}\right)^2+0^2}}, \frac{0}{\sqrt{2^2+\left(-\frac{3}{2}\right)^2+0^2}}$ or, $\frac{2}{5/2}, \frac{-3/2}{5/2}, 0$

or, $\frac{4}{5}, -\frac{3}{5}, 0$

The given line passes through the point having a position vector $\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}$ and is parallel to the vector $\vec{b} = 2\hat{i} - \frac{3}{2}\hat{j} + 0\hat{k}$.

Therefore, it's vector equation is

$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - \frac{3}{2}\hat{j} + 0\hat{k}\right)$

5. p = 2x, q = x

p + q = 1 $\Rightarrow$ 2x + x = 1

$\Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$

$\therefore p = \frac{2}{3}, q = \frac{1}{3}$

P (at least 4 successes) = P (X = 4) + P (X = 5) + P (X = 6)

= C(6, 4) p^4q^2 + C(6, 5) p^5q + p^6

$= C(6,4)\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + C(6,5)\left(\frac{2}{5}\right)^5\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^6$

$= 15\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + 6\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^6$

$= \left(\frac{2}{3}\right)^4\left(5 + 2 \times \frac{2}{3} + \frac{4}{9}\right)$

$= \left(\frac{2}{3}\right)^4\left(\frac{15+12+4}{9}\right)$

$= \left(\frac{2}{3}\right)^4\left(\frac{31}{9}\right) = \frac{496}{729}$

6. Given:

Bag A = (4R + 5B) balls

Bag B = (3R + 7B) balls

Therefore, required probability is given by,

P (balls of same colour) = P (both red) + P (both black)

= $\frac{4}{9} \times \frac{3}{10} + \frac{7}{10} \times \frac{5}{9}$

= $\frac{12}{90} + \frac{35}{90}$

= $\frac{47}{90}$

Section B

7. Let $I = \int \frac{2x+1}{\sqrt{3x+2}} dx$

Let 2x + 1 = λ(3x + 2) + μ

On equating the coefficients of like powers of x on both sides, we get

3λ = 2 and 2λ + μ = 1

$\Rightarrow \quad \lambda=\frac{2}{3}$ and $2\times\frac{2}{3}+\mu=1$

$\Rightarrow \quad \lambda=\frac{2}{3}$ and $\mu=\frac{-1}{3}$

$\therefore \quad I=\int\frac{\lambda(3x+2)+\mu}{\sqrt{3x+2}}dx$

$=\lambda\int\frac{3x+2}{\sqrt{3x+2}}dx+\mu\int\frac{1}{\sqrt{3x+2}}dx$

$=\lambda\int(3x+2)^{\frac{1}{2}}dx+\mu\int(3x+2)^{\frac{-1}{2}}dx$

$=\lambda\times\frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}\times3}+\mu\frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2}\times3}+c$

$=\frac{2}{3}\times\frac{2}{9}\times(3x+2)^{\frac{3}{2}}-\frac{1}{3}\times\frac{2}{3}(3x+2)^{\frac{1}{2}}+c$

$=\frac{4}{27}\times(3x+2)^{\frac{3}{2}}-\frac{2}{9}\times(3x+2)^{\frac{1}{2}}+c$

$=\frac{2}{9}\times\sqrt{3x+2}\left[\frac{2}{3}\times(3x+2)-1\right]+c$

$=\frac{2}{9}\sqrt{3x+2}\left[\frac{6x+4-3}{3}\right]+c$

$=\frac{2}{27}\sqrt{3x+2}(6x+1)+c$

$\therefore \quad I=\frac{2}{27}(6x+1)\sqrt{3x+2}+c$

8. According to the question,we have to solve,

$\cos^2 x\frac{dy}{dx}+y=\tan x$

Therefore,on dividing both sides by $\cos^2$x, the given equation can be rewritten as,

$\frac{dy}{dx}+\frac{y}{\cos^2 x}=\frac{\tan x}{\cos^2 x}$

$\Rightarrow \quad \frac{dy}{dx}+y\cdot\sec^2 x=\tan x\sec^2 x$

which is the linear differential equation of the form

$\frac{dy}{dx}+Py=Q$

Here, P = $\sec^2$x and Q = tanx $\sec^2$x

$\therefore \quad \text{IF}=e^{\int Pdx}=e^{\int\sec^2 xdx}=e^{\tan x}\ \left[\because\int\sec^2 xdx=\tan x+C\right]$

The solution of linear differential equation is given by

$y\times\text{IF}=\int(Q\times\text{IF})dx+C$

$\therefore \quad y\times e^{\tan x}=\int\tan x\sec^2 x\cdot e^{\tan x}dx+C$ ...(i)

Therefore,on putting tan x = t $\Rightarrow$ $\sec^2$x dx = dt in Eq.(i),

we get

$ye^{tanx}=\int\underset{I}{t}\,\underset{II}{e^t}dt+C$

$\Rightarrow \quad ye^{\tan x}=t\int e^t dt-\int\left[\frac{d}{dt}(t)\int e^t dt\right]dt+C$ [using integration by parts]

$\Rightarrow \quad ye^{\tan x}=te^t-\int 1\times e^t dt+C$

$\Rightarrow \quad ye^{\tan x}=te^t-e^t+C$

$\therefore \quad ye^{\tan x}=\tan xe^{\tan x}-e^{\tan x}+C[\because t=\tan x]$

On dividing both sides by $e^{\tan x}$, we get

y = tan x - 1 + $Ce^{-\tan x}$

which is the required solution.

OR

It is clear that if a circle touches y-axis at the origin must have its centre on x-axis, because x-axis being at right angles to y-axis is the normal or line of radius of the circle.

Therefore, the centre of the circle is (r,0) where r is the radius of the circle.

$\therefore$ Equation of the required circle is $(x - r)^2 + (y - 0)^2 = r^2$

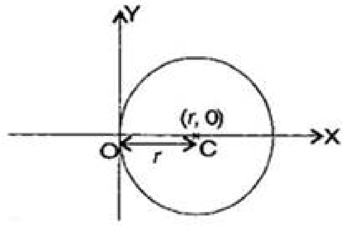
$\Rightarrow x^2+r^2-2rx+y^2=r^2$

$\Rightarrow x^2+y^2=2rx$ ...(i)

Here r is the only arbitrary constant.

$\therefore$ differentiating ....(i) w.r.t. x, we get

$2x+2y\frac{dy}{dx}=2r$ ...(ii)



Putting the value of 2r from eq. (ii) in eq. (i), we get

$x^2+y^2=\left(2x+2y\frac{dy}{dx}\right)x$

$\Rightarrow x^2+y^2=2x^2+2xy\frac{dy}{dx}$

$\Rightarrow -2xy\frac{dy}{dx}-x^2+y^2=0$

$\Rightarrow 2xy\frac{dy}{dx}+x^2-y^2=0$

$\Rightarrow 2xy\frac{dy}{dx}+x^2=y^2$, which is the required differential equation.

9. To prove: $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2\left(|\vec{a}|^2+|\vec{b}|^2\right)$

Now we have, $|\vec{a}+\vec{b}|^2=(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$ $[|\vec{x}|^2=\vec{x}\cdot\vec{x}]$

$\Rightarrow \quad |\vec{a}+\vec{b}|^2=(\vec{a}+\vec{b}).\vec{a}+(\vec{a}+\vec{b}).\vec{b}$ [By distributivity of dot product over vector addition]

$\Rightarrow \quad |\vec{a}+\vec{b}|^2=\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}$ [By distributivity of dot product over vector addition]

$\Rightarrow \quad |\vec{a}+\vec{b}|^2=|\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2$ ....(i) $[\because \vec{a}\cdot\vec{b}=\vec{b}.\vec{a}]$

$|\vec{a}-\vec{b}|^2=(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$ $[|\vec{x}|^2=\vec{x}\cdot\vec{x}]$

$\Rightarrow \quad |\vec{a}-\vec{b}|^2=(\vec{a}+\vec{b}).\vec{a}+(\vec{a}+\vec{b}).\overrightarrow{-b}$ [By distributivity of dot product over vector addition]

$\Rightarrow \quad |\vec{a}-\vec{b}|^2=\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}-\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{b}$ [By distributivity of dot product over vector addition]

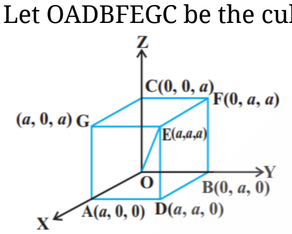
$\Rightarrow \quad |\vec{a}-\vec{b}|^2=|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$ ....(ii) $[\because \vec{a}\cdot\vec{b}=\vec{b}.\vec{a}]$

Adding (i) and (ii), we get

$|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2\left(|\vec{a}|^2+|\vec{b}|^2\right)$

10. A cube is a rectangular parallelopiped having equal length, breadth and height.

Let OADBFEGC be the cube with each side of length a units.



The four diagonals are OE, AF, BG and CD.

The direction cosines of the diagonal OE which is the line joining two points O and E are

$\frac{a-0}{\sqrt{a^2+a^2+a^2}},\frac{a-0}{\sqrt{a^2+a^2+a^2}},\frac{a-0}{\sqrt{a^2+a^2+a^2}}$

i.e., $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

Similarly, the direction cosines of AF, BG and CD are $\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}}$ respectively.

Let l, m, n be the direction cosines of the given line which makes angles $\alpha,\beta,\gamma,\delta$ with OE, AF, BG, CD, respectively. Then

$\cos\alpha=\frac{1}{\sqrt{3}}(l+m+n)$

$\cos\beta=\frac{1}{\sqrt{3}}(-l+m+n)$

$\cos\gamma=\frac{1}{\sqrt{3}}(l-m+n)$

$\cos\delta=\frac{1}{\sqrt{3}}(l+m-n)$

Squaring and adding, we get

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$

= $\frac{1}{3}\left[(l+m+n)^2+(-l+m+n)^2\right]$ + $(l-m+n)^2+(l+m-n)^2\right]$

= $\frac{1}{3}\left[4\left(l^2+m^2+n^2\right)\right]=\frac{4}{3}$ (as l^2 + m^2 + n^2 = 1)

OR

Given equations of lines are,

$\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$.................(i)

and $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$......................(ii)

On comparing above equations with vector

equation $\vec{r}=\vec{a}+\lambda\vec{b}$,we get

$\overrightarrow{a_1}=\hat{i}+\hat{j}=2\hat{i}-\hat{j}+\hat{k},\vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$

and $\overrightarrow{a_2}=2\hat{i}+\hat{j}-\hat{k},\overrightarrow{b_2}=\hat{3i}-5\hat{j}+2\hat{k}$

We know that, the shortest distance between two lines is given by

$d=\left|\frac{\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)\cdot\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right|$............................(iii)

Now, $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&1\\3&-5&2\end{vmatrix}$

$=\hat{i}(-2+5)-\hat{j}(4-3)+\hat{k}(-10+3)$...(iv)

and $\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{(3)^2+(-1)^2+(-7)^2}$

$=\sqrt{9+1+49}=\sqrt{59}$..................(v)

Also, $\overrightarrow{a_2}-\overrightarrow{a_1}=(2\hat{i}+\hat{j}-\hat{k})-(\hat{i}+\hat{j})$

$=\hat{i}-\hat{k}$ ...(vi)

From Eqs. (iii), (iv), (v) and (vi), we get

$d=\left|\frac{(3\hat{i}-\hat{j}-7\hat{k})\cdot(\hat{i}-\hat{k})}{\sqrt{59}}\right|$

$\Rightarrow\quad d=\left|\frac{3-0+7}{\sqrt{59}}\right|=\frac{10}{\sqrt{59}}$

Hence, required shortest distance is $\frac{10}{\sqrt{59}}$ units.

Section C

11. Let $I=\int\frac{dx}{\sin x(3+2\cos x)}$

Put t = cosx

dt = -sinx dx

$\frac{dt}{-\sin x}=dx$

$\frac{dt}{-\sin x}=dx$

$=-\int\frac{dt}{\sin^2 x(3+2t)}=-\int\frac{dt}{(1-\cos^2 x)(3+2t)}$

$=-\int\frac{dt}{(1-t^2)(3+2t)}$

$\frac{1}{(1-t^2)(3+2t)}=\frac{1}{(1-t)(1+t)(3+2t)}$

Now using partial fractions Putting $\frac{1}{(1-t)(1+t)(3+2t)}=\frac{A}{1-t}+\frac{B}{1+t}+\frac{C}{3+2t}$ ... (1)

A(1 + t)(3 + 2t) + B(1 - t)(3 + 2t) + C(1 + t)(1 - t) = 1

Now Putting 1 + t = 0

t = -1

A(0) + B(2)(3 - 2) + C(0) = 1

$B=\frac{1}{2}$

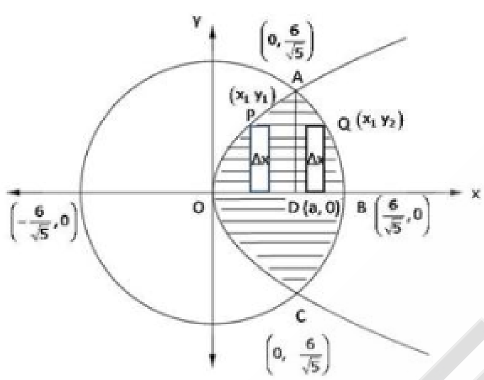
Now Putting 1 - t = 0

t = 1

A(2) (5) + B(0) + C(0) = 1

$A = \frac{1}{10}$
Now Putting 3 + 2t = 0
$t = -\frac{3}{2}$
$A(0) + B(0) + C\left(1 - \frac{9}{4}\right) = 1$
$C = \frac{-4}{5}$
$\frac{1}{(1-t)(1+t)(3+2t)} = \frac{1}{10} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{1+t} - \frac{4}{5} \times \frac{1}{3+2t}$
$\int \frac{1}{(1-t)(1+t)(3+2t)} dt = \frac{1}{10} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{1+t} dt - \frac{4}{5} \int \frac{1}{3+2t} dt$
$= -\frac{1}{10} \log|1-t| + \frac{1}{2} \log|1+t| - \frac{4}{5} \times \frac{\log|3+2t|}{2} + c$
$= -\frac{1}{10} \log|1-\cos x| + \frac{1}{2} \log|1+\cos x| - \frac{2}{5} \log|3+2\cos x| + c$

12. To find area $\left\{(x,y) : y^2 \leq 5x, 5x^2 + 5y^2 \leq 36\right\}$
The curves included are,
$\Rightarrow y^2 = 5x$ ...(i)
$5x^2 + 5y^2 = 36$
$x^2 + y^2 = \frac{36}{5}$ ...(ii)
Equation (i) represents a parabola with vertex (0, 0) and axis as x-axis.
Equation (ii) represents a circle with centre (0, 0) and radius $\frac{6}{\sqrt{5}}$ and meets axes at $\left(\pm\frac{6}{\sqrt{5}}, 0\right)$ and $\left(0, \pm\frac{6}{\sqrt{5}}\right)$, x coordinate of point of intersection of circle and parabola is a where, $a = \frac{-25+\sqrt{1345}}{10}$, A rough sketch of curves is:-



Therefore, we have,
Required area = Region OCBAO
A = 2(Region |OBAO)
= 2 (Region ODAO + Region DBAD)
$= 2\left[\int_0^2 \sqrt{5x}\,dx + \int_2^{\frac{6}{\sqrt{5}}} \sqrt{\left(\frac{6}{\sqrt{5}}\right)^2 - x^2}\,dx\right]$
$= 2\left[\left(\sqrt{5}\frac{2}{3}x\sqrt{x}\right)_0^2 + \left(\frac{x}{2}\sqrt{\left(\frac{6}{\sqrt{5}}\right)^2 - x^2} + \frac{36}{10}\sin^{-1}\left(\frac{x\sqrt{5}}{6}\right)\right)_2^{\frac{6}{\sqrt{5}}}\right.$
$= \frac{4\sqrt{5}}{3}a\sqrt{a} + 2\left\{\left(0 + \frac{18}{5} \cdot \frac{\pi}{2}\right) - \left(\frac{a}{2}\sqrt{\left(\frac{6}{\sqrt{5}}\right)^2 - a^2} + \frac{18}{5}\sin^{-1}\left(\frac{a\sqrt{5}}{6}\right)\right)\right\}$
Thus, $A = \frac{4\sqrt{5}}{a}a^{\frac{3}{2}} + \frac{18\pi}{5} - a\sqrt{\frac{36}{5} - a^2} - \frac{36}{5}\sin^{-1}\left(\frac{a\sqrt{5}}{6}\right)$
Where, $a = \frac{-25+\sqrt{1345}}{10}$

OR

Given circles are $x^2 + y^2 = 4$...(i)
$(x-2)^2 + y^2 = 4$...(ii)
Eq. (i) is a circle with centre origin and
Radius = 2.
Eq. (ii) is a circle with centre C (2, 0) and

Radius = 2.

On solving Eqs. (i) and (ii), we get

$(x-2)^2+y^2=x^2+y^2$

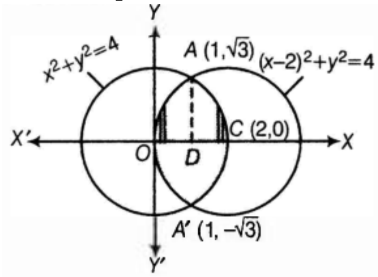
$\Rightarrow x^2-4x+4+y^2=x^2+y^2$

$\Rightarrow x=1$

On putting x = 1 in Eq. (i), we get

$y=\pm\sqrt{3}$

Thus, the points of intersection of the given circles are A (1, $\sqrt{3}$) and A'(1,-$\sqrt{3}$).



Clearly, required area= Area of the enclosed region OACA'O between circles

= 2 [Area of the region ODCAO]

=2 [Area of the region ODAO + Area of the region DCAD]

$=2\left[\int_0^1 y_2 dx+\int_1^2 y_1 dx\right]$

$=2\left[\int_0^1 \sqrt{4-(x-2)^2}\, dx+\int_1^2 \sqrt{4-x^2}\, dx\right]$

$=2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2}+\frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1+2\left[\frac{1}{2}x\sqrt{4-x^2}+\frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$

$=\left[(x-2)\sqrt{4-(x-2)^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1+\left[x\sqrt{4-x^2}+4\sin^{-1}\frac{x}{2}\right]_1^2$

$=\left[\left\{-\sqrt{3}+4\sin^{-1}\left(\frac{-1}{2}\right)\right\}-0-4\sin^{-1}(-1)\right]+\left[0+4\sin^{-1}1-\sqrt{3}-4\sin^{-1}\frac{1}{2}\right]$

$=\left[\left(-\sqrt{3}-4\times\frac{\pi}{6}\right)+4\times\frac{\pi}{2}\right]+\left[4\times\frac{\pi}{2}-\sqrt{3}-4\times\frac{\pi}{6}\right]$

$=\left(-\sqrt{3}-\frac{2\pi}{3}+2\pi\right)+\left(2\pi-\sqrt{3}-\frac{2\pi}{3}\right)$

$=\frac{8\pi}{3}-2\sqrt{3}$ sq units.

13. Given

$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

$\vec{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(-2\hat{i}+3\hat{j}+8\hat{k})$

Here, we have

$\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}$

$\overrightarrow{\mathrm{b}_1}=2\hat{i}+3\hat{j}+6\hat{\mathrm{k}}$

$\overrightarrow{a_2}=3\hat{i}+3\hat{j}-5\hat{k}$

$\overrightarrow{\mathrm{b}_2}=-2\hat{i}+3\hat{j}+8\hat{\mathrm{k}}$

Thus,

$$\overrightarrow{\mathrm{b}_1}\times\overrightarrow{\mathrm{b}_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ -2 & 3 & 8\end{vmatrix}$$

$=\hat{i}(24-18)-\hat{j}(16+12)+\hat{k}(6-6)$

$\overrightarrow{\mathrm{b}_1}\times\overrightarrow{\mathrm{b}_2}=6\hat{\mathrm{i}}-28\hat{\mathrm{j}}+0\hat{\mathrm{k}}$

$\therefore|\overrightarrow{\mathrm{b}_1}\times\overrightarrow{\mathrm{b}_2}|=\sqrt{6^2+(-28)^2+0^2}$

$=\sqrt{36+784+9}$

$=\sqrt{820}$

$\overrightarrow{a_2}-\overrightarrow{a_1}=(3-1)\hat{i}+(3-2)\hat{j}+(-5+4)\hat{\mathrm{k}}$

$\therefore\overrightarrow{a_2}-\overrightarrow{a_1}=2\hat{i}+\hat{j}-\hat{k}$

Now, we have

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (6\hat{i} - 28\hat{j} + 0\hat{k}) \cdot (2\hat{i} + \hat{j} - \hat{k})$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 - 28 + 0

= -16

Thus,the shortest distance between the given lines is

$d = \left| \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right|$

$\Rightarrow d = \left| \frac{-16}{\sqrt{820}} \right|$

$\therefore d = \frac{16}{\sqrt{820}}$ units

CASE-BASED/DATA-BASED

14. Let A be the event of commiting an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

    i. Using Bayes' theorem, we have

    $P(E_1 \mid A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$

    $= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$

    $\therefore$ Required probability $= P(\bar{E}_1 \mid A)$

    $= 1 - P(E_1 \mid A) = 1 - \frac{30}{47} = \frac{17}{47}$

    ii. $\sum_{i=1}^{3} P(E_i \mid A) = P(E_1 \mid A) + P(E_2 \mid A) + P(E_3 \mid A)$

    = 1 [$\because$ Sum of posterior probabilities is 1]